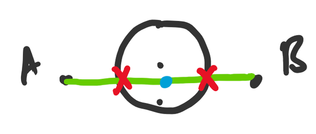
CHOMP

Sunday, August 3, 2025 8:16 PM

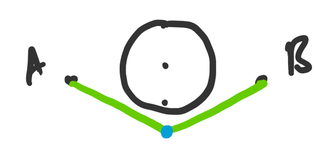
Covariant Hamiltonian Optimization for Motion Planning

Sneak Peek!

Before



After



No collisions!

Core piece

- Starts w/ initial trajectory
- Uses ∇ to pull trajectory out of collision
- simultaneously optimizes joint velocities & accelerations

Objective Function

Minimize $U(\xi) = \underline{F\_obs(\xi)} + \lambda \underline{F\_smooth}(\xi)$

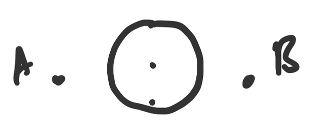
F_smooth - Penalizes high velocities/accelerations Promotes smoothness

F_obs - Penalizes obstacle proximity Promotes Safety

λ: Weight parameter balancing smoothness vs. obstacle avoidance

Hand calculation

0. Setup



$q_0 = (0,0)$

$q_1 = (1,0)$

$q_2 = (2,0)$

Trajectory

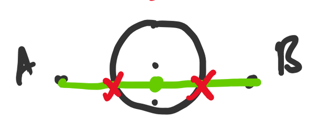
Fixed

1. Initialize

Create initial Trajectory

$\xi_0 = [(0,0), (1,0), (2,0)]$

collision!



$d = \sqrt{(1-1)^2 + (0-.2)^2} = .2$ Euclidean distance to object center

$d = .2 < .3$ ← object radius

We prove this point mathematically has collided

2. Define cost func.

A. $c(x,y) = \begin{cases} \text{high cost}, & \text{if inside obstacle} \\ \text{low cost}, & \text{if near obstacle} \\ 0, & \text{if far from obstacle} \end{cases}$

specifically

$c(d) = \begin{cases} (-d+.5)^2 & \text{if } d < .3 \\ 0 & \text{if } d \geq .3 \end{cases}$ inside obstacle / outside obstacle

$d = .2$

$c(.2) = (-.2 + .5)^2 = .09 < .3$, so inside!

B. Smoothness cost func

$F\_smooth = \frac{1}{2}[v_1^2 + v_2^2]$ (ΔX_{1-2}, ΔX_{2-3})

$v_1 = q_1 - q_0 = (1,0) - (0,0) = (1,0)$

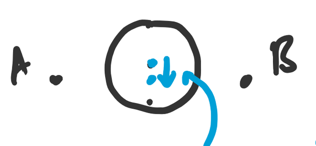
$v_2 = q_2 - q_1 = (2,0) - (1,0) = (1,0)$

$F\_smooth = \frac{1}{2}[(1^2+0^2) + (1^2+0^2)] = \frac{1}{2}[2] = 1$

3. Calculate the Gradients

for $q_1 = (1,0)$

A. Obstacle cost gradient



Find direction & magnitude of this vector

$(1,0) - (1,.2) = (0,-.2)$

unit vector $= \frac{(0,-.2)}{.2} = (0,-1)$ Direction

$c(d) = (-d+.5)^2$

$\frac{dc(d)}{dd} = -2(-d+.5) = 2d - 1 = .4 - 1 = -.6$

$\nabla F\_obs = \nabla_d C \cdot \vec{u} = -.6 \cdot (0,-1)$

$\nabla F\_obs = (0,.6)$

B. Smoothness Cost Gradient

$\nabla F\_smooth = 2q_1 - q_0 - q_2$

$= (2,0) - (0,0) - (2,0)$

$= 0,0$

$2q_1 = 2(1,0) = (2,0)$

$q_0 = (0,0)$

$q_2 = (2,0)$

Logically makes sense b/c currently in a straight line

4. Combine Gradients

$\nabla U(\xi) = \lambda \nabla F\_smooth + \nabla F\_obs$

setting $\lambda = 1$ for now...

$\nabla U(\xi) = \nabla F\_smooth + \nabla F\_obs$

$= (0,0) + (0,.6) = (0,.6)$

Update the waypoint...

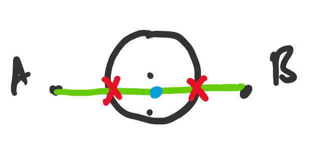
$q_1 = q_1 - \eta \nabla U$

Set $\eta = .5$ & plug in

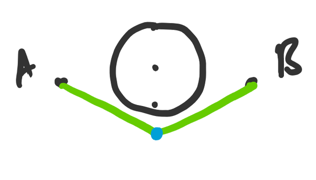
$q_1 = (1,0) - .5(0,.6)$

$q_1 = (1,-.3)$

Before



After



No collisions!