

Diffusion Policy

- Core idea:
- Diffusion Policy applies diffusion models (from img generation) to generate robot trajectories & control policies
 - Model learns to refine random noise into coherent action sequence by iteratively denoising

Simple Example (1D)

Step 1 - Setup State Space

$\begin{bmatrix} 1 \\ S \end{bmatrix} \quad G$
 $S = [0, 10]$
Action space = $[-5, +5]$ Continuous Action Space
Trajectory = $[a_i]$ Timesteps
Goal: Generate trajectory $0 \rightarrow 3$
so 1 timestep
Perfect Trajectory = $[+3.0]$

Simplified to $S()$

Neural Net: $f(\text{noisy_trajectory, timestep, current_state}) \rightarrow \text{predicted_noise}$
 $L = ||\hat{\epsilon}^{NT} - \epsilon||^2$ (MSE) CS

Training Phases

Step 1: Collect goal train data

Sample 1	Sample 2	Sample 3
Initial State: 0.0	0.0	0.0
Goal: 3.0	3.0	3.0
Goal Trajectory: $[+3.0]$	$[+2.8]$	$[+3.3]$
Result: $0.0 + 3.0 = 3$	$0 + 2.8 = 2.8$	$0 + 3.3 = 3.3$

Step 2: Forward Diffusion (Training adding noise)

- Noise schedule
- timestep 0: No noise
- 1: $x_0 + N(0, \sigma=0.3)$
- 2: $x_1 + N(0, \sigma=0.7)$
- 3: $x_2 + N(0, \sigma=1.5)$

For $[+3.0]$ NN training $([NT, T, CS], \text{truncnoise})$
t=0 (clean) $x_0 = [+3.0]$
t=1 (light noise) $x_1 = x_0 + N(0, 0.3^2)$ $([+3.2], 1, 0.0], [+0.2])$
 $x_1 = [+3.2]$ "When I see $[+3.2]$ at t=1 w/ $CS=0.0$, the noise is $[+0.2]$ "
t=2 (medium) $x_2 = x_1 + N(0, 0.7^2)$ $([+3.7], 2, 0.0], [+0.5])$
 $x_2 = [+3.7]$ "When I see $[+3.7]$ at t=2 w/ $CS=0.0$, the noise is $[+0.5]$ "
t=3 (heavy) $x_3 = x_2 + N(0, 1.5^2)$ $([+4.8], 3, 0.0], [+1.1])$
 $x_3 = [+4.8]$ "When I see $[+4.8]$ at t=3 w/ $CS=0.0$, the noise is $[+1.1]$ "

Generation Phase

Step 1 - Start w/ noise
 $CS = 0.0$ (Start position)
Goal: 3.0

Random starting trajectory: $x_3 = [+5.1]$
Would result in $0.0 + 5.1 = 5.1 >> 3.0$ overshoot!

Step 2 - Reverse Diffusion (Remove Noise Iteratively)

t=3 \rightarrow 2 (Remove heavy noise)
Noisy-trajectory: $[+5.1]$
timestep: 3
current state: 0.0
 $S([+5.1, 3, 0.0]) = [+1.1]$ (predicted noise)
 $x_2 = x_1 - \text{noise} = [+5.1] - [+1.1] = [+4.0]$

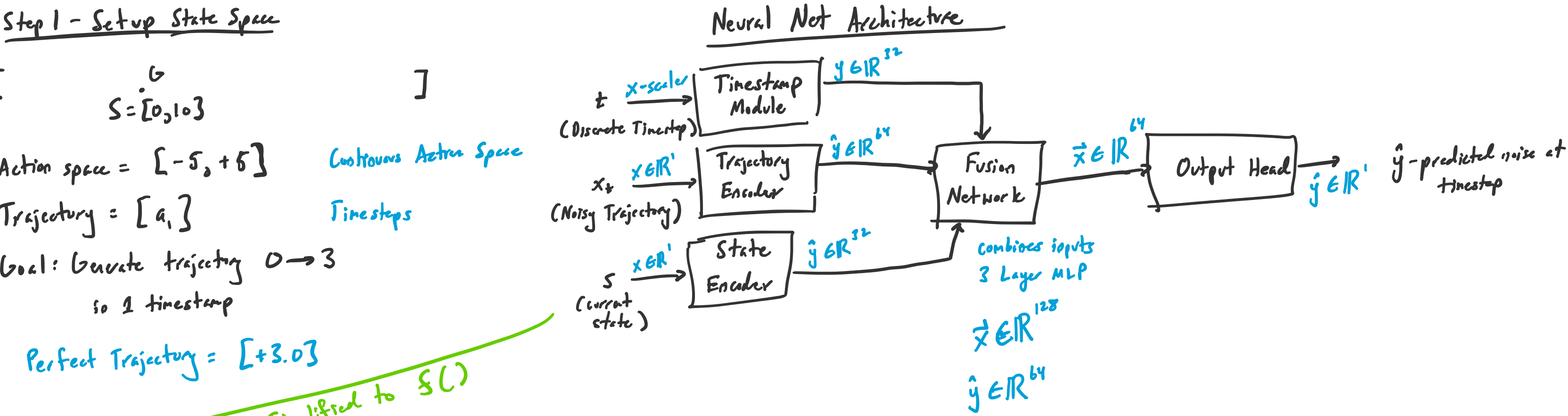
t=2 \rightarrow 1 (Remove medium noise)
 $S([+4.0, 2, 0.0]) = [+0.5]$
 $x_1 = x_2 - \text{noise} = [+4.0] - [+0.5] = [+3.5]$

t=1 \rightarrow 0
 $S([+3.5, 1, 0.0]) = [+0.2]$
 $x_0 = x_1 - \text{noise} = [+3.5] - [+0.2] = [+3.3]$

Goal Check: $|3.3 - 3.0| = .3$ Very close!

Execution Phase

generated trajectory = $[+3.3]$
execute $[+3.3]$
As per:
State: 3.3
Goal: 3.0
Goal Achieved! (otherwise repeat!)



- Timestep Module - Convert discrete timestep to continuous
- Trajectory Encoder - Process noisy action sequence
- State Encoder - Process the current robot current state & goal
- Fusion Network - combine all variable inputs
- Output Head - Predict noise to remove