

# Diffusion Policy

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## Diffusion Policy

### Core idea:

- Diffusion Policy applies diffusion models (from ing generation) to generate robot trajectories & control policies
- Model learns to refine random noise into coherent action sequence by iteratively denoising

### Simple Example (1D)

#### Step 1 - Setup State Space

$S \in \mathbb{R}^G$   
 $S = [0, 10]$   
Action space =  $[-5, +5]$  Continuous Action Space  
Trajectory =  $[a,]$  Timesteps  
Goal: Generate trajectory  $0 \rightarrow 3$   
so 1 timestep  
Perfect Trajectory =  $[+3.0]$

Neural Net:

$$f(\text{noisy-trajectory}, \text{timestep}, \text{current-state}) \rightarrow \text{predicted-noise}$$
$$L = \|\hat{z} - z\|_2^2 \quad (\text{MSE})$$

#### Training Phases:

##### Step 1: Collect goal train data

| sample 1         | Sample 1        | Sample 2        | Sample 3        |
|------------------|-----------------|-----------------|-----------------|
| Initial State:   | 0.0             | 0.0             | 0.0             |
| Goal:            | 3.0             | 3.0             | 3.0             |
| Goal Trajectory: | $[+3.0]$        | $[+2.8]$        | $[+3.3]$        |
| Result:          | $0.0 + 3.0 = 3$ | $0 + 2.8 = 2.8$ | $0 + 3.3 = 3.3$ |

#### Step 2: Forward Diffusion (Training adding noise)

##### Noise schedule

- timestep 0: No noise
- 1:  $x_0 + N(0, \sigma=0.3)$  Add noise from normal distribution This makes it easier for the NN's to learn the noise
- 2:  $x_1 + N(0, \sigma=0.7)$
- 3:  $x_2 + N(0, \sigma=1.5)$

For  $[+3.0]$  NN training  $([NT, T, CS], \text{true noise})$

$t=0$  (clean)

$x_0 = [3.0]$

$t=1$  (light noise)

$$x_1 = x_0 + N(0, \sigma^2) = ([+3.2], 1, 0.0], [+0.2])$$

"When I see  $[+3.2]$  at  $t=1$  w/  $CS=0.0$ , the noise is  $[+0.2]$ "

$t=2$  (medium)

$$x_2 = x_1 + N(0, \sigma^2) = ([+3.7], 2, 0.0], [+0.5])$$

"When I see  $[+3.7]$  at  $t=2$  w/  $CS=0.0$ , the noise is  $[+0.5]$ "

$t=3$  (heavy)

$$x_3 = x_2 + N(0, \sigma^2) = ([+4.8], 3, 0.0], [+1.1])$$

"When I see  $[+4.8]$  at  $t=3$  w/  $CS=0.0$ , the noise is  $[+1.1]$ "

#### Generation Phase

##### Step 1 - Start w/ noise

CS - 0.0 (start position)

Goal: 3.0

Random starting trajectory:  $x_0 = [+5.1]$

Would result in  $0.0 + 5.1 = 5.1 > 3.0$  overshoot!

##### Step 2 - Reverse Diffusion (Remove Noise iteratively)

$t=3 \rightarrow 2$  (Remove heavy noise)

Noisy-trajectory:  $[+5.1]$

timestep: 3

current state: 0.0

$$f([+5.1], 3, 0.0) = [+1.1] \quad (\text{predicted noise})$$

$$x_2 = x_1 - \text{noise} = [+5.1] - [+1.1] = [+4.0]$$

$t=2 \rightarrow 1$  (Remove medium noise)

$$f([+4.0], 2, 0.0) = [+0.5]$$

$$x_1 = x_2 - \text{noise} = [+4.0] - [+0.5] = [+3.5]$$

$t=1 \rightarrow 0$

$$f([+3.5], 1, 0.0) = [+0.2]$$

$$x_0 = x_1 - \text{noise} = [+3.5] - [+0.2] = [+3.3]$$

Goal Check:  $1.3 - 3.0 = -1.7$  very close!

#### Execution Phase

generated trajectory =  $[+3.3]$

execute  $[+2.3]$

Assume  $x_{t-1} = 3.3$

state: 3.0

loc: 3.0

Goal Achieved! (otherwise repeat!)

