MPPI

Wednesday, August 6, 2025 7:04 PM

Model Predictive Path Integral

Core Idea:

- Generate random control sequences
- Simulate where this would lead
- Evaluate costs
- Combine best ones using weighted averaging

Simple Example (1D) (NO OBSTACLE)

Weighting: $w_i = \exp(-c_i/\lambda)$

System: $x_{t+1} = x_t + u_t$ Position updates by control input

Cost: $(x_{final} - 5)^2 + .1u^2$ Penalize distance from goal + control effort

Parameters: $k=3$ samples, $H=2$ time steps, $\lambda = 1$ ← Temp

Step 1: Random control sequence & computation

$u_1 = [2,1]$	$u_2 = [3,2]$	$u_3 = [1,4]$	
$x_0 = 0$	$x_0 = 0$	$x_0 = 0$	Initial state
$x_1 = 0+2 = 2$	$x_1 = 0+3 = 3$	$x_1 = 0+1 = 1$	Apply system equation
$x_2 = 2+1 = 3$	$x_2 = 3+2 = 5$	$x_2 = 1+4 = 5$	
$c = (3-5)^2 + .1(2^2+1^2) = 4 + .5 = 4.5$	$c = (5-5)^2 + .1(3^2+2^2) = 0 + 1.3 = 1.3$	$c = (5-5)^2 + .1(1^2+4^2) = 0 + 1.7 = 1.7$	Apply cost function
$w_1 = \exp(-4.5) = .011$	$w_2 = \exp(-1.3) = .273$	$w_3 = \exp(-1.7) = .183$	Calculate weights $\exp(-c)$
$w_1 = .024$	$w_2 = .584$	$w_3 = .392$	Normalize $(w_i / \sum_{i=1}^{N} w_i)$

Step 2: Compute optimal first control

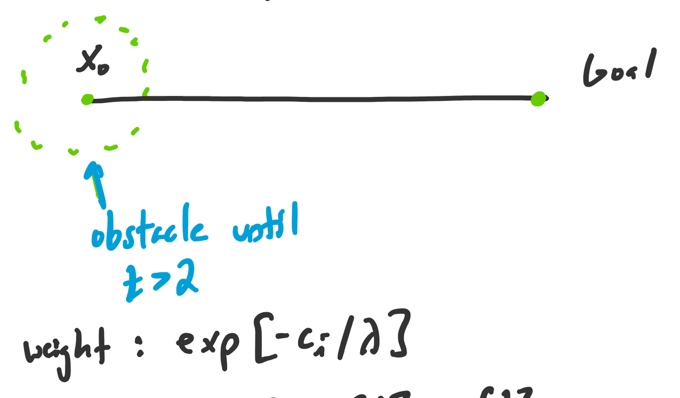
$$u^* = \sum_{i=1}^{3} w_i u_i = (.024)(2) + (.584)(3) + (.392)(1) = 2.19$$

Step 3: Replan given $u[0] = 2.19$

This allows MPPI to adjust to stochastic world

etc. etc.

OK, that was pretty easy. How about we introduce an obstacle



weight: $\exp[-c_i/\lambda]$

System: $x[t+1] = x[t] + u[t]$

Cost: $c = \underbrace{(x_f - 5)^2}_{\text{Penalize not reaching goal}} + \underbrace{\frac{1}{10}\sum u[t]^2}_{\text{Penalize too much effort}} + \underbrace{1000(u[0]^2 + u[1]^2)}_{\text{Penalize moving at } t=0,\ t=1 \text{ (obstacle)}}$

$k=3$ samples, $H=3$ steps, $\lambda=1$

Calculation

$u_1 = [.1, .2, 4.7]$	$u_2 = [0, 0, 5.0]$	$u_3 = [.5, -.2, 4.7]$	
$x_0 = 0$	$x_0 = 0$	$x = 0$	Start at 0
$x_1 = 0 + .1 = .1$	$x_1 = 0+0 = 0$	$x_1 = 0 + .5 = .5$	Calculate future states
$x_2 = .1 + .2 = .3$	$x_2 = 0+0 = 0$	$x_2 = .5 + (-.2) = .3$	
$x_3 = .3 + 4.7 = 5$	$x_3 = 0+5 = 5$	$x_3 = .3 + 4.7 = 5$	
$c_1 = (5-5)^2 + \frac{1}{10}(.1^2 + .2^2 + 4.7^2) + 1000(.1^2 + .2)^2 = 52.214$	$c_2 = (5-5)^2 + \frac{1}{10}(0^2+0^2+5^2) + 1000(0^2+0^2) = 2.5$	$c_3 = (5-5)^2 + \frac{1}{10}(.5^2 + (-.2)^2 + 4.7^2) + 1000(.5^2 + (-.2)^2 + 4.7^2) = 292.24$	Cost function
$w_1 = \exp(-52.214) \simeq 0$	$w_2 = \exp(-2.5) = .082$	$w_3 = \exp(-292.238) \simeq 0$	Calculate weights
$w_1 = 0$	$w_2 = 1.0$	$w_3 = 0$	Normalize weights

Compute u^* (recommended step)

$$u[0]^* = (0)(.1) + (1)(0) + (0)(.5) = 0$$

Very Good! Exactly what I expected

Replan at $t=1$

- Need to decide whether
  - Fixed relative horizon → Always plan 3 steps ahead of $t \in \mathbb{R}$
  - Finite Deadline – Plan 2 steps ahead b/c $\text{count}(t) = 3$

keeping all things the same as $t=0$...

$u_1 = [0, 2.5, 2.5]$	$u_2 = [0, 1, 4]$	$u_3 = [.1, 2.0, 2.9]$
$\vec{x} = [0, 2.5, 5]$	$\vec{x} = [0, 1, 5]$	$\vec{x} = [.1, 2.1, 5]$
$c_1 = 1.25$	$c_2 = 1.7$	$c_3 = 11.242$
$w_1 = .287$	$w_2 = .183$	$w_3 \simeq 0$
$w_1 = .611$	$w_2 = .389$	$w_3 = 0$

Normalized

$u^*[1] = (.611)(0) + (.389)(0) = 0$

$u^*[2] = (.611)(2.5) + (.389)(1) = 1.917$ ← MPPI getting ready to move

$t=2$

$u_1 = [2.0, 2.0, 1.0]$	$u_2 = [1.5, 1.5, 2.0]$	$u_3 = [3.0, 1.0, 1.0]$
$\vec{x} = [2, 4, 5]$	$\vec{x} = [1.5, 3.0, 5.0]$	$\vec{x} = [3.0, 4.0, 5.0]$
$c_1 = .9$	$c_2 = .85$	$c_3 = 1.1$
$w_1 = .407$	$w_2 = .427$	$w_3 = .333$
$w_1 = .349$	$w_2 = .366$	$w_3 = .285$

$u^*[2] = (.349)(2) + (.366)(1.5) + (.285)(3) = 2.102$ } MPPI moves now with obstacle gone