

Core Idea

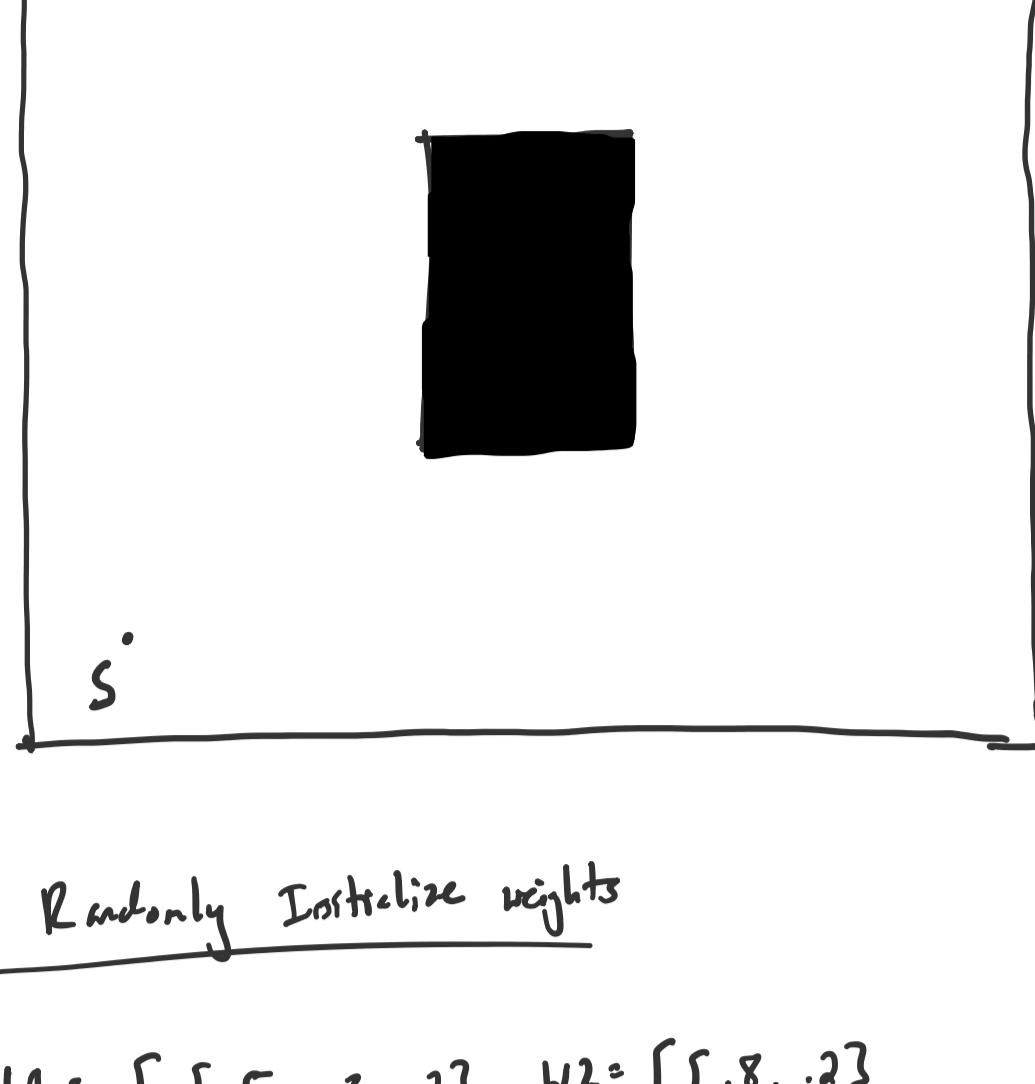
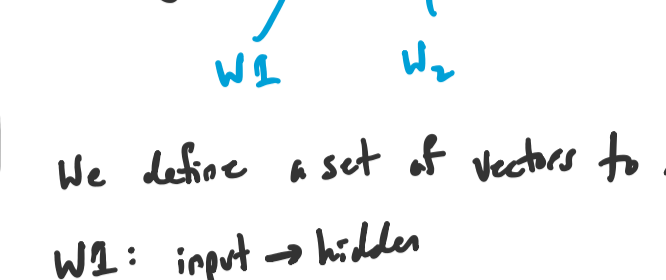
Neural RRT* is Neural Network + RRT*

- RRT* randomly samples points to build its tree
- if we are smart about where we sample, RRT* becomes much better
- if we are smart about which connections to explore, RRT* can converge to solution / optimal solution faster

The Neural Net!

Step 1 - Initialization

Start = (1,1) | Goal = (8,8)

A. Neural Net Definition

We define a set of vectors to store weights

W1: input \rightarrow hiddenW2: hidden \rightarrow output

b1: hidden bias

b2: output bias

B. Randomly Initialize weights

$$W1 = \begin{bmatrix} 1.5 & -3 & .23 \\ -.1 & .4 & -.13 \\ -.2 & .3 & .63 \\ .4 & -.2 & .17 \end{bmatrix} \quad W2 = \begin{bmatrix} 1.8 & .23 \\ -.1 & .73 \\ .3 & -.43 \end{bmatrix}$$

$$b1 = [1.1, -.2, .3] \quad b2 = [0, 0]$$

C. Define the forward pass eqns

$$h_1 = \text{relu}(W1[0,0] \cdot x_1 + W1[1,0] \cdot x_2 + W1[2,0] \cdot x_3 + W1[3,0] \cdot x_4 + b1[0])$$

$$h_2 = \text{relu}(W1[0,1] \cdot x_1 + W1[1,1] \cdot x_2 + W1[2,1] \cdot x_3 + W1[3,1] \cdot x_4 + b1[1])$$

$$h_3 = \text{relu}(W1[0,2] \cdot x_1 + W1[1,2] \cdot x_2 + W1[2,2] \cdot x_3 + W1[3,2] \cdot x_4 + b1[2])$$

* $\forall h_i$: if < 0 , then set = 0

$$y_1 = W2[0,0] \cdot h_1 + W2[1,0] \cdot h_2 + W2[2,0] \cdot h_3 + b2[0]$$

$$y_2 = W2[0,1] \cdot h_1 + W2[1,1] \cdot h_2 + W2[2,1] \cdot h_3 + b2[1]$$

D. Define Gradient Descent Functions

$$L = \frac{1}{2} [(\hat{y}_1 - y_1)^2 + (\hat{y}_2 - y_2)^2] \leftarrow \text{basic MSE}$$

$$\frac{\partial L}{\partial y_1} = \hat{y}_1 - y_1 \quad \frac{\partial L}{\partial y_2} = \hat{y}_2 - y_2$$

$$\frac{\partial L}{\partial W2[i,0]} = (\hat{y}_1 - y_1) \cdot h_{i+1}$$

$$\frac{\partial L}{\partial W2[i,1]} = (\hat{y}_2 - y_2) \cdot h_{i+1}$$

$$\frac{\partial L}{\partial b_2[0]} = \hat{y}_1 - y_1 \quad \frac{\partial L}{\partial b_2[1]} = \hat{y}_2 - y_2$$

$$\frac{\partial L}{\partial h_1} = (\hat{y}_1 - y_1) \cdot W2[0,0] + (\hat{y}_2 - y_2) \cdot W2[0,1]$$

$$\frac{\partial L}{\partial h_2} = (\hat{y}_1 - y_1) \cdot W2[1,0] + (\hat{y}_2 - y_2) \cdot W2[1,1]$$

$$\frac{\partial L}{\partial h_3} = (\hat{y}_1 - y_1) \cdot W2[2,0] + (\hat{y}_2 - y_2) \cdot W2[2,1]$$

$$\frac{\partial L}{\partial W1[i,j]} = \frac{\partial L}{\partial h_{i+1}} \cdot x_{j+1}$$

$$\frac{\partial L}{\partial b1[i]} = \frac{\partial L}{\partial h_{i+1}}$$

$$\frac{\partial L}{\partial h_1} = (\hat{y}_1 - y_1) \cdot W2[0,0] + (\hat{y}_2 - y_2) \cdot W2[0,1]$$

$$\frac{\partial L}{\partial h_2} = (\hat{y}_1 - y_1) \cdot W2[1,0] + (\hat{y}_2 - y_2) \cdot W2[1,1]$$

$$\frac{\partial L}{\partial h_3} = (\hat{y}_1 - y_1) \cdot W2[2,0] + (\hat{y}_2 - y_2) \cdot W2[2,1]$$

$$\frac{\partial L}{\partial W1[i,j]} = \frac{\partial L}{\partial h_{i+1}} \cdot x_{j+1}$$

$$\frac{\partial L}{\partial b1[i]} = \frac{\partial L}{\partial h_{i+1}}$$

E. Weight update rules

$$W2[i,j] = W2[i,j] - \alpha \cdot \frac{\partial L}{\partial W2[i,j]}$$

$$W1[i,j] = W1[i,j] - \alpha \cdot \frac{\partial L}{\partial W1[i,j]}$$

$$b2[i] = b2[i] - \alpha \cdot \frac{\partial L}{\partial b2[i]}$$

$$b1[i] = b1[i] - \alpha \cdot \frac{\partial L}{\partial b1[i]}$$

Step 2: Iteration 1A. Analyse Tree

- coverage - bottom left of grid
- 1 node (1,1)
- Distance to Goal: $d = \sqrt{(8-1)^2 + (8-1)^2} = 9.9$

B. Determine NN input

$$[\text{goal-x}, \text{goal-y}, \text{start-x}, \text{start-y}]$$

$$[8, 8, 1, 1]$$

C. NN Forward pass

Using calculator...

$$\hat{y} = [4.55, .79] \leftarrow A$$

D. Run RRT*

- Query point - [A]
- Candidate Nodes - [S]
- $d = \sqrt{(4.55-1)^2 + (.79-1)^2} = 3.56$
- [S(3.56)] \rightarrow so S is closest
- Check for collision \rightarrow NO collision \checkmark

Step 3 - Iteration 2A. Analyse Tree

- Nodes: 2 [A(7.99), S(9.90)]

B. NN input

- Use closest Node for expansion (A)

$$[8, 8, 4.55, .79]$$

C. NN Forward Pass

$$\hat{y} = [4.487, .562]$$

* But check - this is a bad guess!

D. Run RRT*

- Find Nearest Node \rightarrow A
- [A(2.26), S(3.46)]
- Collision check: NO collision

E. RRT* rewire

- would connecting (4.487, .562) via other nodes be more optimal?
- via (1,1)
- $d = 0 + 3.46 + 3.46$
- Lower! Rewire
- via (4.55, .79) (current)
- $d = 3.56 + .26 = 3.82$

F. Performance Assessment

$$[A(2.99), B(8.25), S(9.9)]$$

Further

NN will not be able to reach goal...

Step 4 - TrainingA. Collect Bad suggestions

Training Sample:

$$\text{input state: } [8, 8, 4.55, .79]$$

$$\hat{y} = [4.487, .562]$$

$$y = [7, 2] \quad \text{we choose this ground truth}$$

B. Calculate Loss

$$L = \frac{1}{2} [(\hat{y}_1 - y_1)^2 + (\hat{y}_2 - y_2)^2]$$

$$L = 4.318$$

C. Backprop

$$\alpha = .01$$

This isn't a backprop tutorial, so... after the updates we re-run the same point

Initial prediction: [4.487, .562]

Epoch 1: [8.665, 2.745] closer

Epoch 2: [6.89, 1.78] VERY close!

Step 5 - Repeat

- It's clear now how this works
- once the net is trained, it will react appropriately to the given situation it's in

